

# DEEP LEARNING MADE EASY WITH TERSE NETWORKS

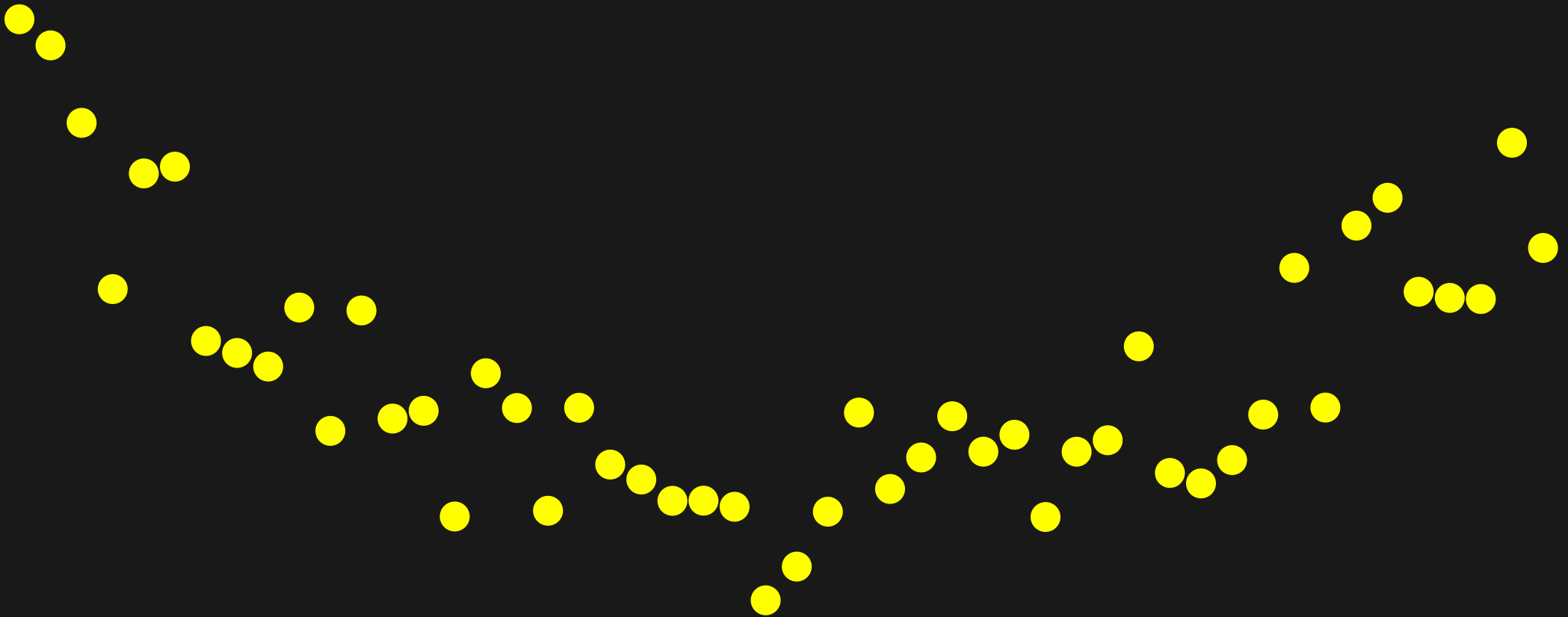
Sebastian Vermeulen

EIPC 2020

Econometrics: Can we recognize  $f$ ?

Machine learning: Can we approximate  $f$ ?

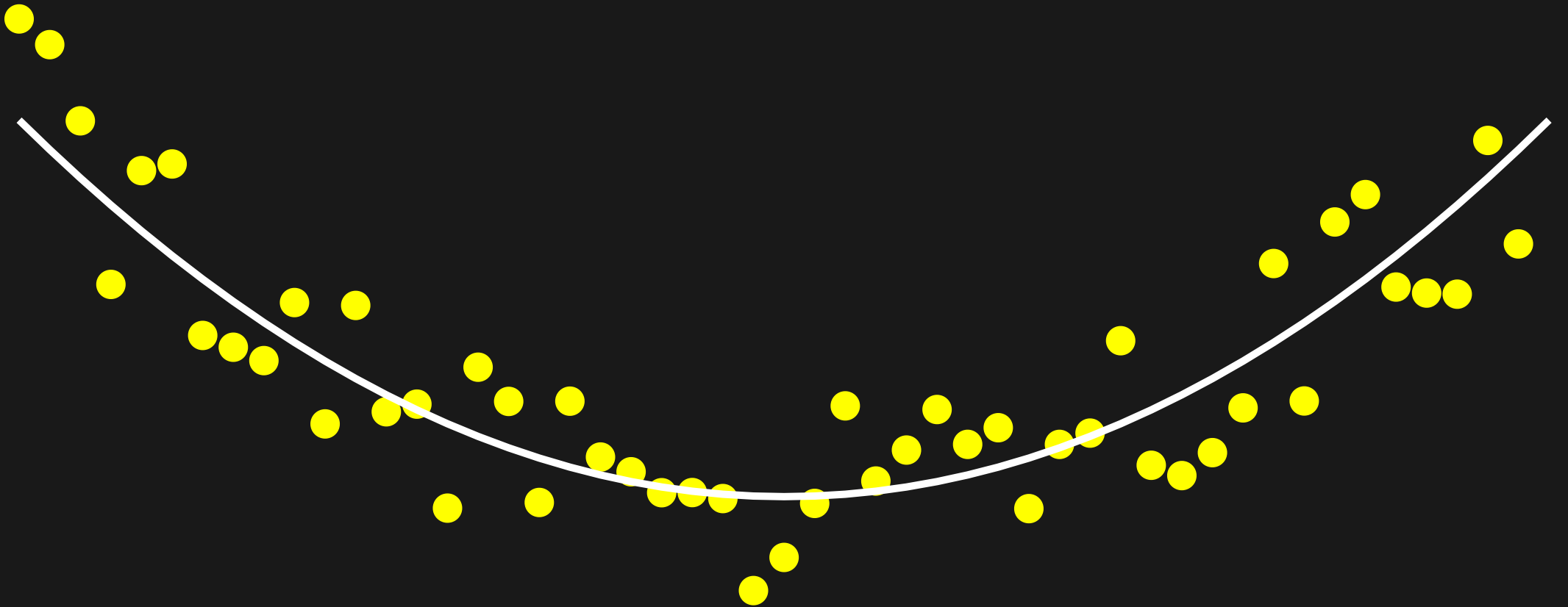
$$y = f(x) + \epsilon$$



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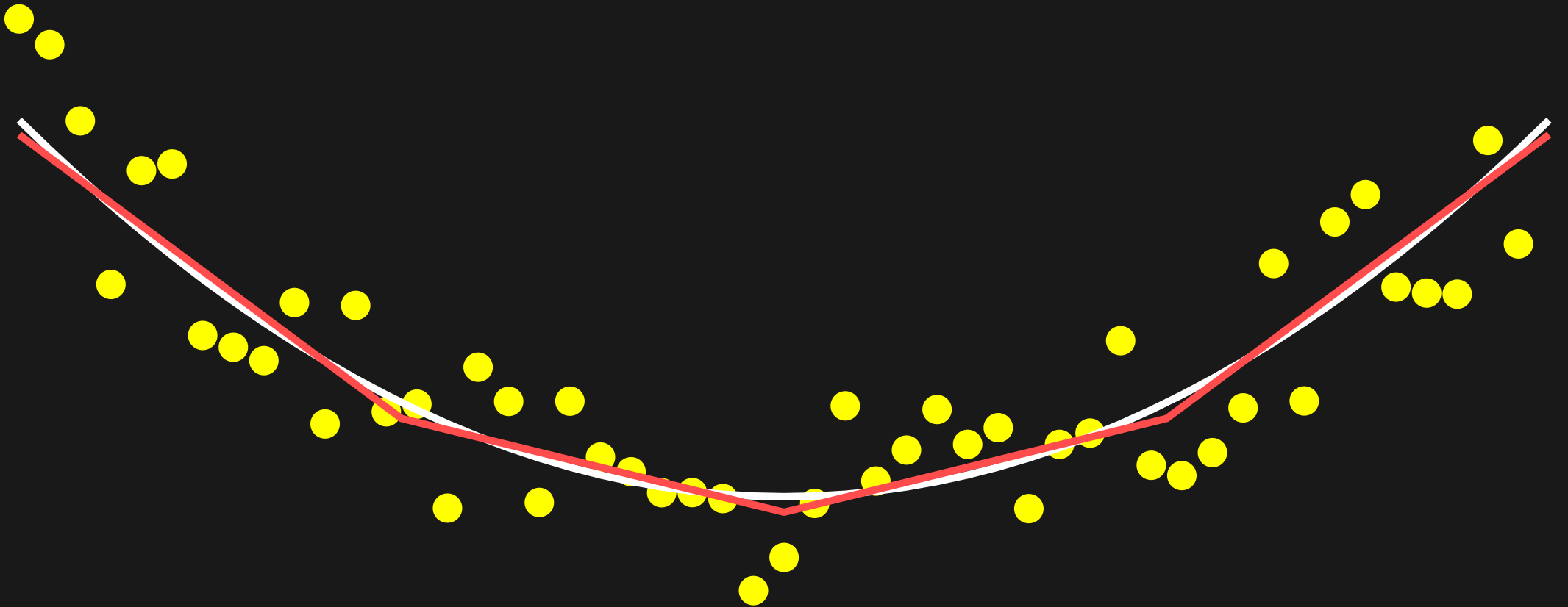
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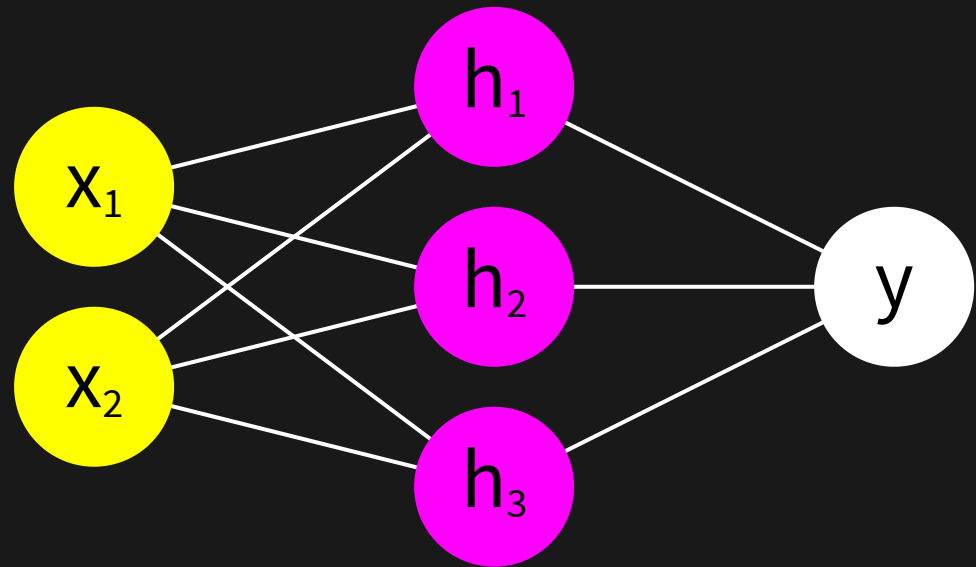
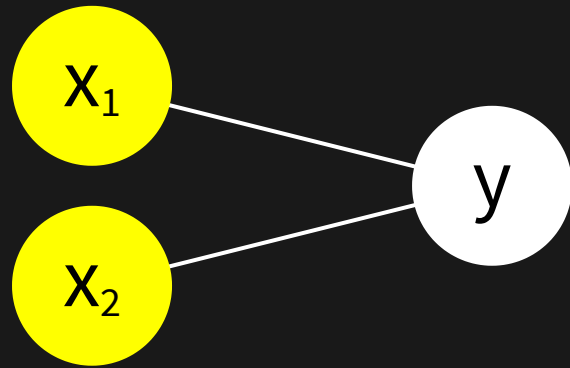
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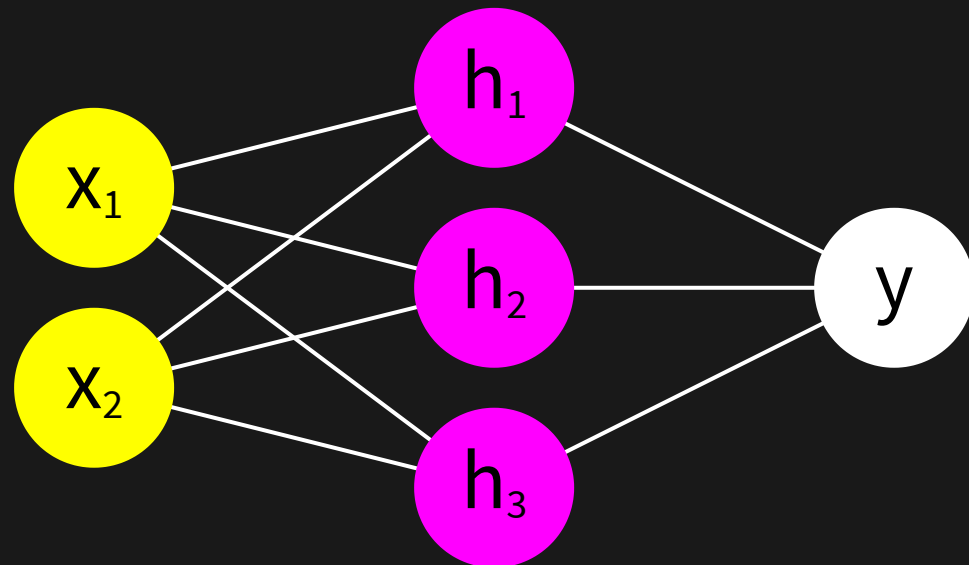
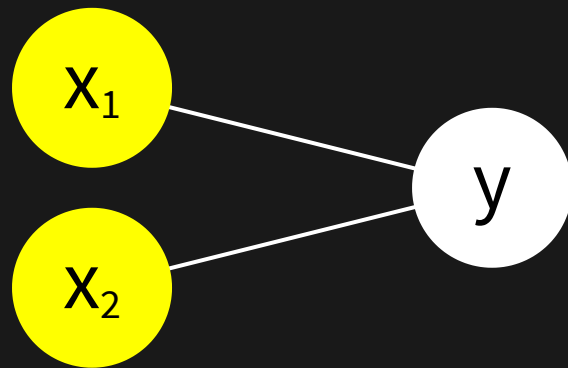
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# Neural Networks



# Neural Networks

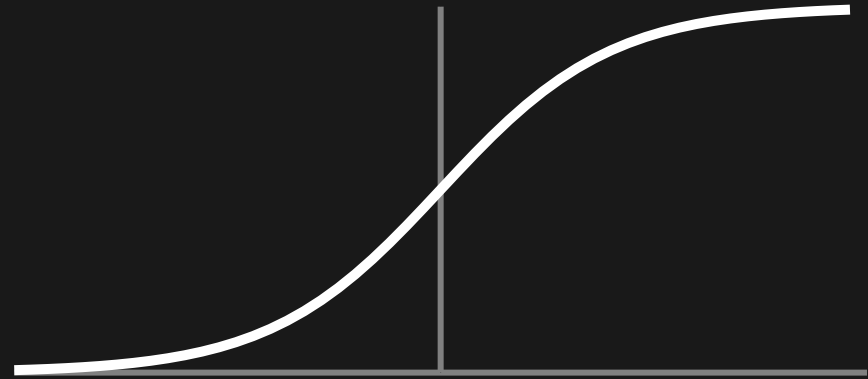
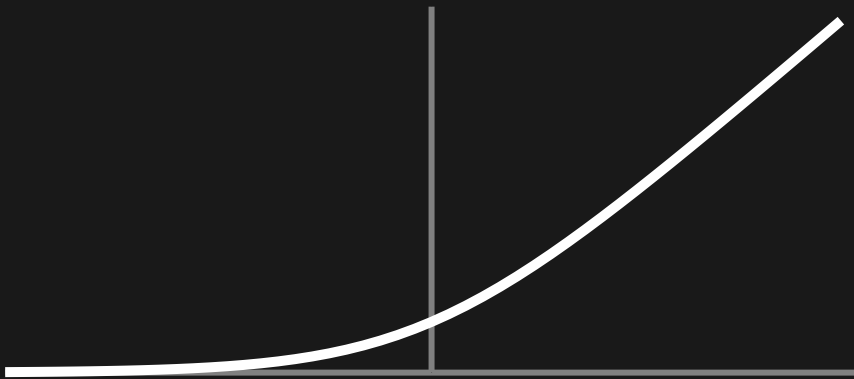
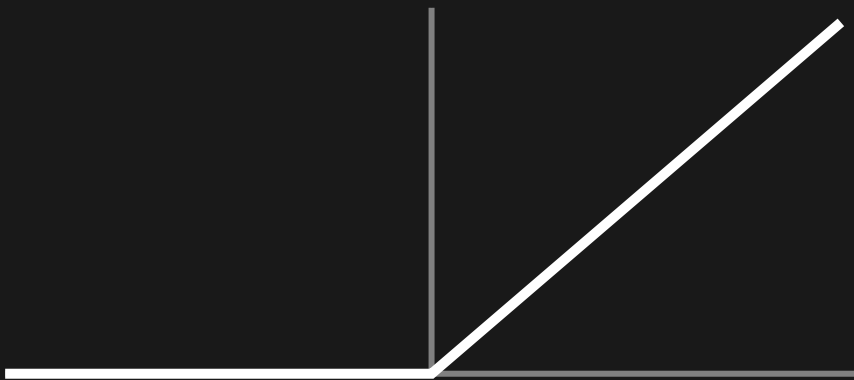


$$h_1(x_t) = \begin{bmatrix} u(a_{11} + x_t' b_{11}) \\ \dots \\ u(a_{n_1 1} + x_t' b_{n_1 1}) \end{bmatrix}$$

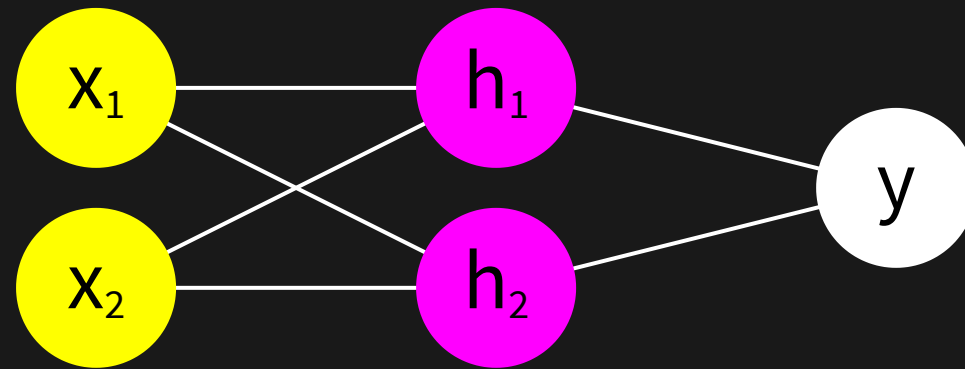
$$h_k(x_t) = \begin{bmatrix} u(a_{1k} + h_{k-1}(x_t)' b_{1k}) \\ \dots \\ u(a_{n_k k} + h_{k-1}(x_t)' b_{n_k k}) \end{bmatrix}$$

$$y_t = h_k(x_t)' \beta$$

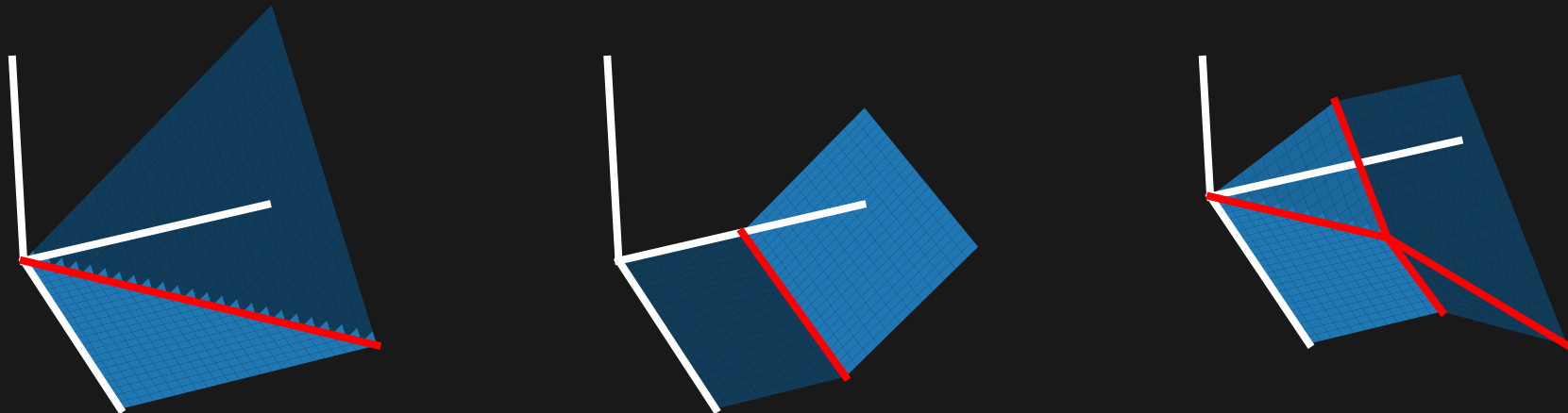
# Activation functions



# Partitioning the 'input space'



$$(x_2 - x_1)^+ - (x_2 - 0.5)^+$$





## Redundancy in the network formula

- Every node is scale-invariant
- Every layer is permutation-invariant

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**TERSENET: DEVOID OF SUPERFLUITY**

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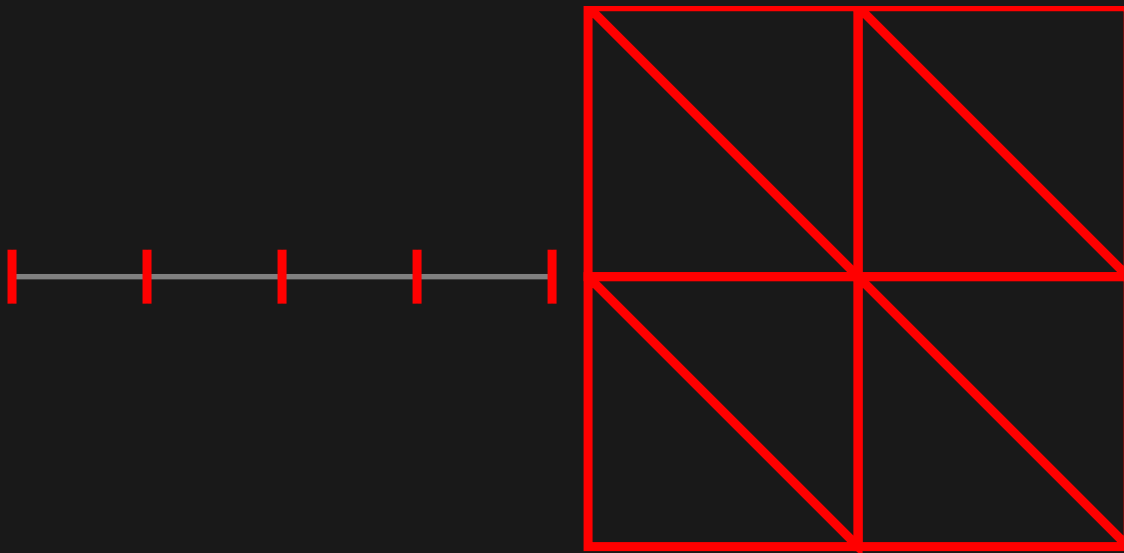


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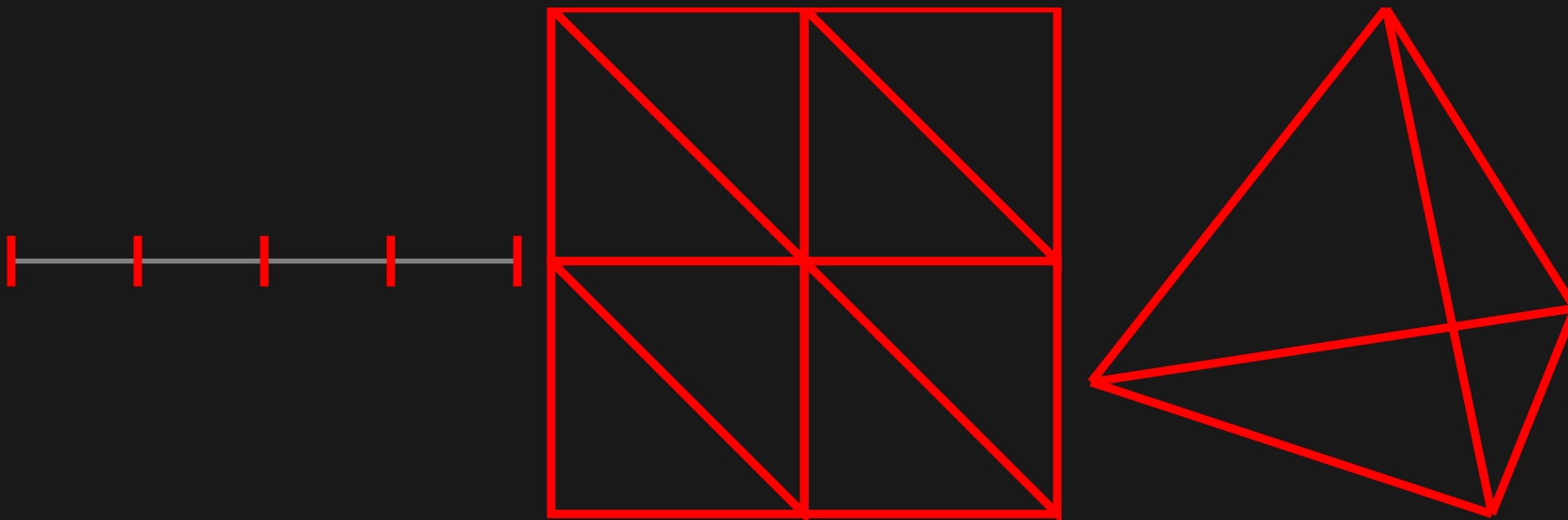


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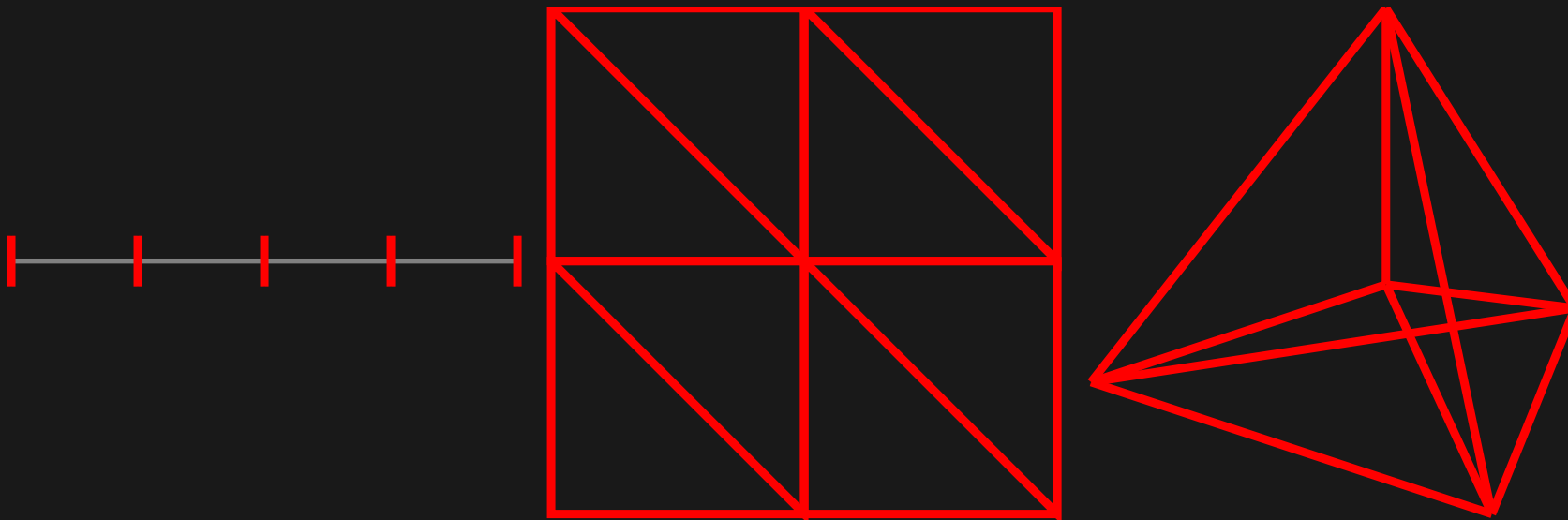


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# TERSENET IS LIKE A LINEAR MODEL:

$$h_{1,i+dj}(x_t) = (j - n(x_{t,i})^+)^+$$

$$h_{2,k}(x_t) = \left( 1 - \sum_{i=1}^d (j_i - n(x_{t,i})^+)^+ \right)^+, \quad j_i = w(k, i)$$

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(Regularized) least squares with OLS or LARS

# PERFORMANCE (THEORETICAL)

- TERSEnet is parameter efficient for  $C^2$  functions

$$\|f - g\|_{\infty} = O(W^{-2/d})$$

- TERSEnet parameters have unique global (easy) optimum

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Insights have wider applications